Department of Curriculum and Instruction

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Tips for Teachers: Making Math Meaningful

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ACPS Alexandria City Public Schools

Mathematics Through Problem Solving vs. Mathematics for Problem Solving:

A Message from Terri H. Mozingo, CAO

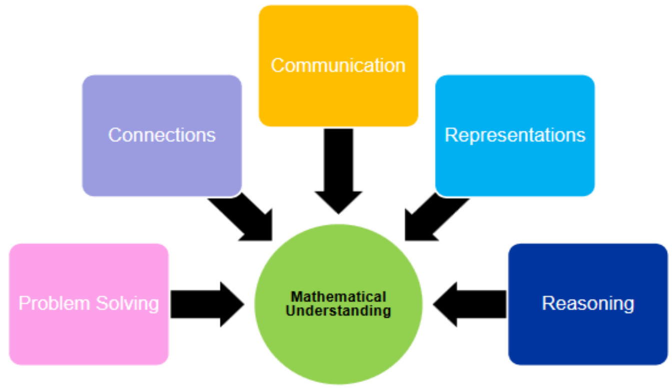
Today, mathematics calls for a focus on sense making, reasoning, and making connections to real-world situations. Students need knowledge and skills that prepare them to **apply mathematics in a variety of contexts**, including their future lives as responsible citizens.

To support teachers and administrators in this process, we are including a series of best practices that will support student understanding in key SOL achievement areas and "**high-yield strategies**."

The ideas included in this edition focus on ways all teachers can enhance student performance in implementing the ***Virginia Mathematics Process Goals.***

They will also revisit key strategies and best practices that current data suggest should be addressed.

The Virginia Mathematics Process Goals play an instrumental role in the teaching and learning of mathematics with understanding. The **VDOE** encourages students to engage in the following practices:



1. **Problem Solving:** A major goal is to help students become competent mathematical problem solvers. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types.
2. **Connections:** Through the application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines.
3. **Communication:** Representing, discussing, reading, writing, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied.
4. **Representations:** Students should move easily among different representations – graphical, numerical, algebraic, verbal, and physical – and recognize that representation is both a process and a product.
5. **Reasoning:** Students should apply sense-making strategies to justify thinking and evaluate conclusions by reasoning from a variety of representations.

More information about the process goals can be found on the **VDOE's website.**

Engaging ALL Learners in Word Problems

Many students struggle with traditional word problems because the **context is sometimes unrealistic**. The traditional word problem can be routine, creating a predictable pattern for solving. These problems usually present students with given information followed by a question students are supposed to answer using the given information.

Traditional Word Problem:

Neela is making rectangular place mats that are 12 inches wide and 15 inches long. What is the least amount of ribbon that she will need to create a ribbon border around 1 place mat?

The following **modifications to the traditional word problem** will help build student perseverance, critical thinking skills, and transfer to real-world problem solving.

1. **Headless Problems:** In headless word problems, students are presented with a question, but the information needed to answer the question is omitted. The students are then tasked with determining what information is needed to answer the given question.

   Example:

   What is the least amount of ribbon Neela will need to create a ribbon border for a place mat?

2. **Tailless Problems**: In tailless word problems, students are given information, asked to make observations about the information, and then tasked with creating questions that can be answered using the given information.

   Example:

   Neela is making 1 rectangular place mat that is 12 inches wide and 15 inches long.

3. **Numberless Problems:** Numberless word problems require students to process what the problem is and what is needed to solve it. This process supports students in understanding the meaning of the context before solving for an answer.

   Example:

   Neela is making rectangular place mats that have a length and a width. What is the least amount of ribbon that she will need to create a ribbon border around one place mat?

   Step 1: Students will process problem without numbers.
   Step 2: Teachers will supply snippets in information.
   Step 3: Teachers will add quantities to math story.

"Every time a student makes a mistake in math, they grow a synapse. … When we make a mistake our brains spark and grow. Mistakes are not only opportunities for learning, as students consider the mistakes, but also times when our brains grown, even if we don't know we have made a mistake."

-Jo Boaler, Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching.

"Five suggestions that can work to open mathematics tasks and increase their potential for learning: Open up the task so that there are multiple methods, pathways, and representations. Include inquiry opportunities. Ask the problem before teaching the method. Add a visual component and ask students how they see the mathematics. Extend the task to make it lower floor and higher ceiling. Ask students to convince and reason; be skeptical."

-Jo Boaler, *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages, and Innovative Teaching*

Supporting Math Understanding Through C-R-A

Concrete Representational Abstract (CRA) is a highly effective instructional approach for developing a deep understanding of math concepts. Students may need instruction through different steps no matter the grade they are in.

Step 1 **Concrete:** The concrete stage is the **"doing"** stage and involves students in physically manipulating objects to solve a math problem.

Step 2 **Representational:** The representational (semi-concrete) stage is known as the **"seeing"** stage. This stage involves using images to represent objects to solve math problems. It is also important to provide an opportunity to bridge the concrete stage and the representational stage to achieve this student's would build it with manipulatives and then draw it.

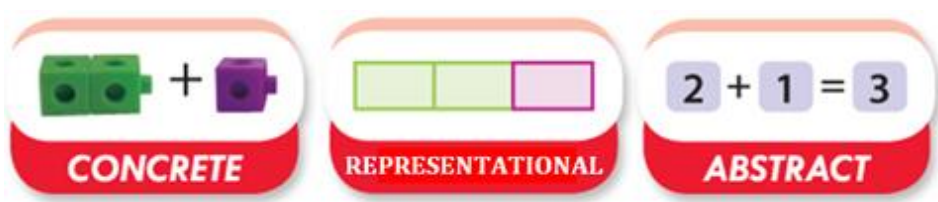
Step 3 **Abstract:** The final stage is the abstract stage, also known as the **"symbolic"** stage. This stage involves using only numbers and symbols to solve a math problem. As there is a need for bridging between concrete and representational, there is also a need for bridging learning between representational and abstract. To do this, student's would draw their thinking and then represent it with numbers and symbols.

CRA is a gradual systematic approach. Each stage builds on to the previous stage and must be taught in sequence.

Steps to implementing CRA:

1. Teach a math concept using **manipulatives.**
2. Allow many opportunities for students to practice the concept using a **variety of manipulatives.**
3. Make sure students **understand the concept** at the concrete level before moving on to the representational level and provide bridging.
4. Introduce **pictures** to represent objects and model the concept.
5. Provide plenty of **time** for student to practice the concept using **drawing or virtual images.**
6. Check **student understanding**. Do not move to the abstract if students haven't mastered the representational level.
7. Teach students to the concept with only **numbers and symbols**. Model the concept.
8. Provide plenty of opportunities for students to **practice using only numbers and symbols.**
9. **Check student understanding**. If students are struggling, go back to the concrete and representational levels.
10. Once the concept is mastered at the abstract level, periodically bring back the concept for students to **practice**.

Remember that modeling the concept and providing lots of opportunities to engage with the concept are important at all three levels. Students need time to make connections and build on what they already know.



Productive Struggle Is Okay!

It's uncomfortable to struggle, but struggling is an important part of learning. Productive struggle is not about being frustrated. **Students need to develop critical thinking skills and perseverance to be able to take on a challenging problem through different strategies and approaches.** Students need to know that it is okay to not know the answer and be willing to participate in the inquiry process to uncover strategies to solve a problem.

One way to support students in productive struggle is by **building a mathematical mindset community**. Here are the components of a Mathematical Mindset Community from ***youcubed* at Stanford University:**

- Teachers and students believe everyone can learn math at **HIGH LEVELS**.
- The math is **VISUAL.**
- The environment is filled with **wonder and curiosity**.
- **Communication and connections** are valued.
- The math is **OPEN.**
- The classroom is a **risk-taking, mistake-valuing environment.**

A second way to support students in productive struggle is by having students engage in learning activities that are challenging and promote inquiry. ***Desmos*** has developed guiding principles for developing great math activities. Below are a few of those guiding principles

- **Create an intellectual need for new mathematical skills:** Ask yourself, "Why did a mathematician invent the skill I'm trying to help students learn? What problem were they trying to solve? How did this skill make their intellectual life easier?" Then ask yourself, "How can I help students experience that need?" We calculate because calculations offer more certainty than estimations. We use variables so we don't have to run the same calculation over and over again. We prove because we want to settle some doubt. Before we offer the aspirin, we need to make sure students are experiencing a headache.
- **Create problematic activities:** A problematic activity feels *focused* while a problem-free activity *meanders*. A problem-free activity picks at a piece of mathematics and asks lots of small questions about it, but the larger frame for those smaller questions isn't apparent.

  A problem-free task gives students a parabola and then asks questions about its vertex, its line of symmetry, and its intercepts, simply because it *can* ask those questions, not because it *must*.

  Don't create an activity with lots of small pieces of analysis at the start that are only clarified by some larger problem later. Help us understand why we're here. Give us the larger problem now.
- **Connect representations:** Understanding the connections between representations of a situation – tables, equations, graphs, and contexts – helps students understand the representations themselves.

  In a typical word problem, the student converts the context into a table, equation, or graph, and then translates between those three formats, leaving the context behind. Re-connect the math to the context.

***"In mathematics the art of proposing a question must be held of higher value than solving it."* Georg Cantor**

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